Ph.D. Defense

Market Design for Integrated Energy Systems of the Future

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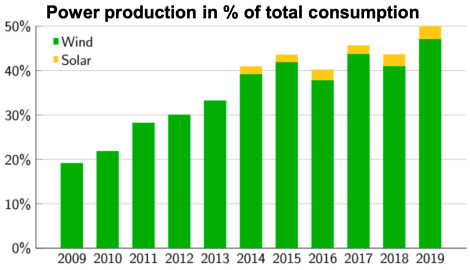
Supervisors: **Pierre Pinson**, **Jalal Kazempour**, and **Ana Virag**

June 1, 2022

Technical University of Denmark

Energy systems are evolving: Green transition

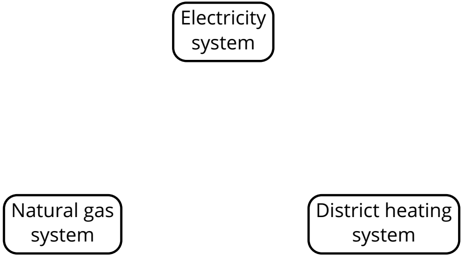
For example, in Denmark:



Source: Green Power Denmark

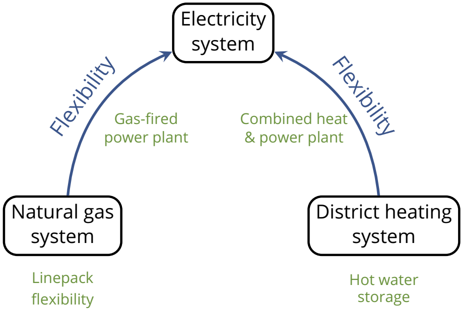
- Fossil fuels $\rightarrow$ **weather-dependent renewables**
- High **uncertainty** and **variability**
- Additional **operational flexibility** needed

Energy systems are evolving: Growing interdependence



Energy systems are evolving: Growing interdependence

- Operational synergies → **cross-carrier flexibility**
  - flexible operation of **boundary agents**
  - **network flexibility** from short-term storage



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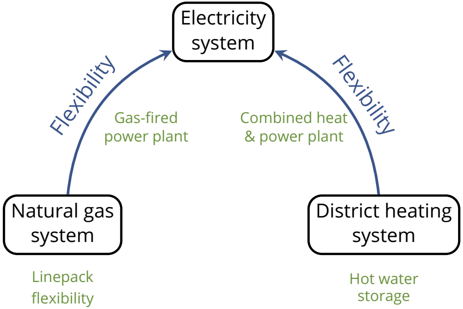
- Sequential and separate energy markets $\rightarrow$ **over-/under-estimation** of flexibility



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- Operational synergies → **cross-carrier flexibility**
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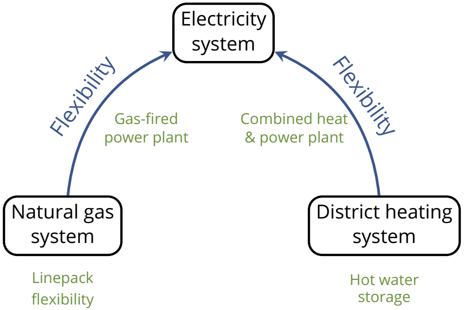
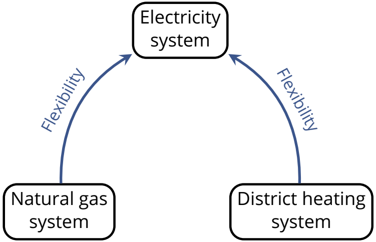
- Sequential and separate energy markets → **over-/under-estimation** of flexibility
- **Market-based coordination** is crucial



Research questions

1. How to design **generic** and **efficient** market mechanisms and products to harness cross-carrier flexibility?
2. How to model and mitigate **uncertainty propagation** among energy systems via market-based coordination?



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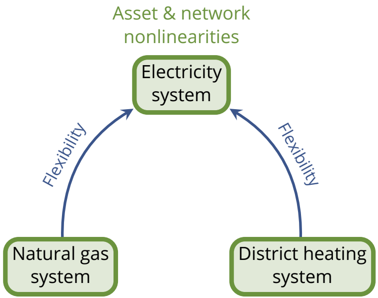

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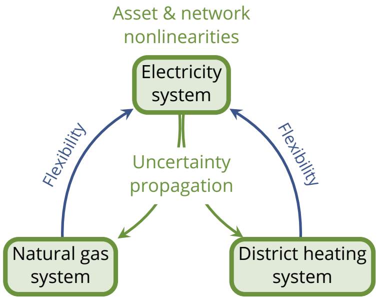

Towards flexibility-centric electricity markets

- **Spatial price equilibrium** using linear programming (LP)

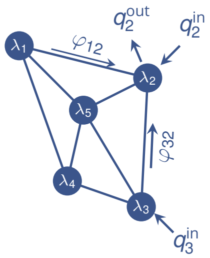

Figure inspiration: V. Dvorkin, Stochastic & private energy system optimization, 2021.

Towards flexibility-centric electricity markets

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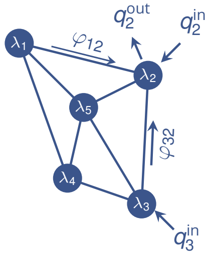
- LP is limiting, as **nonlinearities** common to:
  - costs and constraints of market participants
  - physical flow models in networks
  - uncertainty modeling approaches



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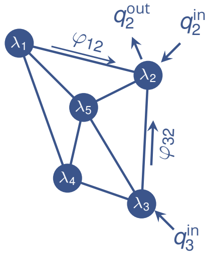

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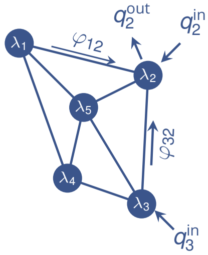

Objective 1

To develop a **general flexibility-centric** electricity market framework which admits nonlinearities in uncertainty, assets, and energy networks.

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Thesis contributions: Objective 1

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1. A **multi-period** & **multi-commodity** conic electricity market
   - Asset and network nonlinearities as **second-order cone (SOC)** constraints
   - Variety of **flexibility services** as additional commodities
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   - No properties lost in moving from linear towards **conic** markets

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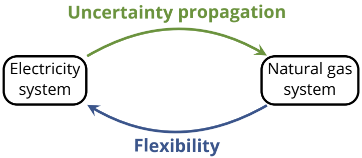
   - No properties lost in moving from linear towards **conic** markets
3. Optimally-sized **policy-based reserves** over capacity-based reserves
   - Lower operations cost with **guarantees against uncertainty realizations**

Towards uncertainty-aware energy system coordination

- Uncertainty in natural gas systems → **price spikes**, **network congestion**



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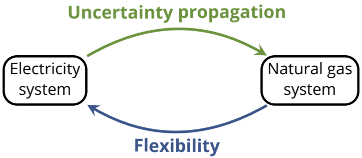

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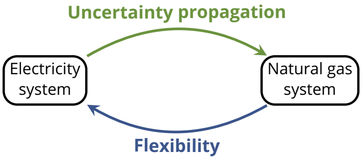

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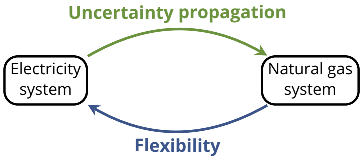

Objective 2

To develop a methodology to harness cross-carrier flexibility in energy markets while taking **uncertainty propagation** into account.

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1. **Uncertainty-aware** electricity and gas dispatch with linepack flexibility
   - Nonlinear and non-convex gas flow dynamics under uncertainty
   - Trade-off: operations cost vs. robustness to **uncertainty propagation**

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   - Analytical description of system state to the uncertainty propagated
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   - Market-based **minimization of variance** of state variables
3. Efficient **pricing scheme** to **remunerate** (**penalize**) agents for **mitigating** (**aggravating**) uncertainty and variance

Publications

Flexibility-centric electricity markets:

1. A. Ratha, P. Pinson, H. Le Cadre, A. Virag and J. Kazempour, "**Moving from linear to conic markets for electricity**", submitted to ***European Journal of Operational Research***, (under review, second round), 2021.

2. A. Ratha, J. Kazempour, A. Virag and P. Pinson, "**Exploring market properties of policy-based reserve procurement for power systems**", in ***2019 IEEE 58th Conference on Decision and Control (CDC)***, Nice, pp. 7498-7505.

Uncertainty-aware coordination among energy systems:

3. A. Ratha, A. Schwele, J. Kazempour, P. Pinson, S. Shariat Torbaghan and A. Virag, "**Affine policies for flexibility provision by natural gas networks to power systems**", in ***Electric Power Systems Research***, Volume 189, Article 106565, December 2020.

4. V. Dvorkin, A. Ratha, P. Pinson and J. Kazempour, "**Stochastic control and pricing for natural gas networks**", in ***IEEE Transactions on Control of Network Systems***, Volume 9, Issue 1, pp. 450-462, March 2022.

Outline

Preliminaries

Conic market

A market-clearing problem that admits **convex strategy sets** of market participants involving **second-order cones** of arbitrary dimensions.

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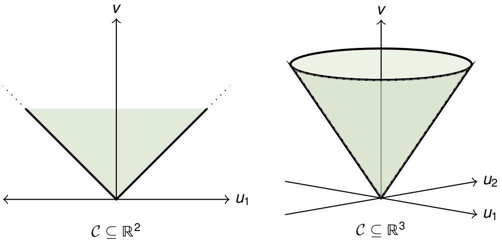
Conic market

A market-clearing problem that admits **convex strategy sets** of market participants involving **second-order cones** of arbitrary dimensions.

Second-order cone (SOC)

A SOC $\mathcal{C}$ of dimension m is a convex set defined, for tuple $(\mathbf{u}, v)$, $\mathbf{u} \in \mathbb{R}^m$ and $v \in \mathbb{R}_+$, as

$$\mathcal{C} := \left\{ \begin{bmatrix} \mathbf{u} \\ v \end{bmatrix} \;\middle|\; \|\mathbf{u}\| \leqslant v \right\} \subseteq \mathbb{R}^{m+1}.$$



Market setting

- **Hourly day-ahead** market cleared over $T = 24$ hours
- P commodities of two kinds: **energy** and **flexibility services**
- Heterogeneous participants, $i \in \mathcal{I}$

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- P commodities of two kinds: **energy** and **flexibility services**
- Heterogeneous participants, $i \in \mathcal{I}$
- Participant's decision vector $\mathbf{q}_{it} \in \mathbb{R}^{K_i}$, where $K_i \geqslant P$

$$\mathbf{q}_i = \begin{bmatrix} \mathbf{q}_{i1} \\ \mathbf{q}_{i2} \\ \vdots \\ \mathbf{q}_{iT} \end{bmatrix} \in \mathbb{R}^{K_i T}$$

- **Temporally-separable** convex quadratic cost function, $c_{it}(\mathbf{q}_{it}) : \mathbb{R}^{K_i} \mapsto \mathbb{R}$:

$$c_{it}(\mathbf{q}_{it}) = {\mathbf{c}^{\mathrm{L}}_{it}}^\top \mathbf{q}_{it} + {\mathbf{q}_{it}}^\top \operatorname{diag}(\mathbf{c}^{\mathrm{Q}}_{it}) \, \mathbf{q}_{it}$$

SOC constraints

Generic SOC constraint

A generic SOC constraint on variable $\mathbf{q}_i$ of i-th market participant is

$$\|\mathbf{A}_i \mathbf{q}_i + \mathbf{b}_i\| \leqslant \mathbf{d}_i^\top \mathbf{q}_i + e_i \qquad .$$

Parameters $\mathbf{A}_i \in \mathbb{R}^{m_i \times K_i T}$, $\mathbf{b}_i \in \mathbb{R}^{m_i}$, $\mathbf{d}_i \in \mathbb{R}^{K_i T}$ and $e_i \in \mathbb{R}$ embody the **structural** and **geometrical** information of each constraint.

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- Special cases:
  - $\mathbf{A}_i = \mathbf{0} \implies 0 \leqslant \mathbf{d}_i^\top \mathbf{q}_i + e_i$ **(linear inequalities)**
  - $\mathbf{d}_i = \mathbf{0},\ e_i \geqslant 0 \implies \|\mathbf{A}_i \mathbf{q}_i + \mathbf{b}_i\| \leqslant e_i$ **(quadratic inequalities)**

Conic market as an optimization problem

$$\min_{\mathbf{q}_i,\, \mathbf{z}_i} \quad \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} z_{it} + {\mathbf{c}_{it}^{\mathrm{L}}}^{\top} \mathbf{q}_{it}$$

$$\text{s.t.} \quad \|\mathbf{C}_{it}^{\mathrm{Q}}\, \mathbf{q}_{it}\|^2 \leq z_{it},\ \forall t,\ \forall i \qquad :(\mu_{it}^{\mathrm{Q}}, \kappa_{it}^{\mathrm{Q}}, \nu_{it}^{\mathrm{Q}})$$ **Objective reformulation**

$$\|\mathbf{A}_{ij}\, \mathbf{q}_i + \mathbf{b}_{ij}\| \leqslant \mathbf{d}_{ij}^{\top}\, \mathbf{q}_i + e_{ij},\ j \in \mathcal{J}_i,\ \forall i \qquad :(\mu_{ij},\ \nu_{ij})$$ **Multiple SOC constraints**

$$\mathbf{F}_i\, \mathbf{q}_i = \mathbf{h}_i,\ \forall i \qquad :(\gamma_i)$$ **Equality constraints**

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Multiple SOC constraints

$$\mathbf{F}_i\, \mathbf{q}_i = \mathbf{h}_i,\ \forall i \qquad :(\gamma_i)$$

Equality constraints

$$\sum_{i\in\mathcal{I}_n} \mathbf{G}_{ip}\, \mathbf{q}_{ip} = \mathbf{0}_T,\ \forall p,\ \forall n \qquad :(\lambda_{pn})$$

Supply-demand balance

$$-\overline{\mathbf{s}} \leqslant \sum_{n\in\mathcal{N}} \mathbf{\Psi}_{(:,n)} \left(\sum_{i\in\mathcal{I}_n} \sum_{p\in\mathcal{P}} [\mathbf{G}_{ip}\, \mathbf{q}_{ip}]_t \right) \leqslant \overline{\mathbf{s}},\ \forall t \qquad :(\underline{\varrho}_t,\ \overline{\varrho}_t)$$

Network flow constraints

Bid structure

Bid structure

Conic market bids

Participant i located at network node n_i submits a bid

$$\mathcal{B}_i := \left(n_i,\ \{\mathbf{A}_{ij}, \mathbf{b}_{ij}, \mathbf{d}_{ij}, e_{ij}\}_{j \in \mathcal{J}_i},\ \mathbf{F}_i, \mathbf{h}_i,\ \{\mathbf{G}_{ip}\}_{p \in \mathcal{P}},\ \{\mathbf{c}^{\mathrm{Q}}_{it}, \mathbf{c}^{\mathrm{L}}_{it}\}_{t \in \mathcal{T}}\right).$$

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  - enable trades in energy and multiple **flexibility services**
  - admit **quadratic costs** without linear approximations

Spatial price equilibrium underlying the conic market

centrally-solved market-clearing optimization $\iff$

spatial price equilibrium

For each participant, *i*:
max. Revenue - Cost

Network operator's problem:
max. Congestion rent

Market clearing conditions:
Supply = Demand, $\forall p \in \mathcal{P}$
Net nodal injection = Energy transported, $\forall n \in \mathcal{N}$

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- Desired **economic properties** proven analytically:
  1. **efficiency** of the market
  2. **cost recovery** of participants
  3. **revenue adequacy** of the market operator

Use Case: Uncertainty-aware electricity markets

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- Sequential energy and reserve markets

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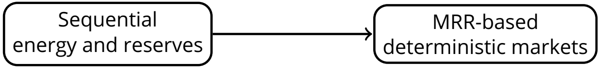
- **Deterministic** linear markets with **minimum reserve requirements (MRR)**



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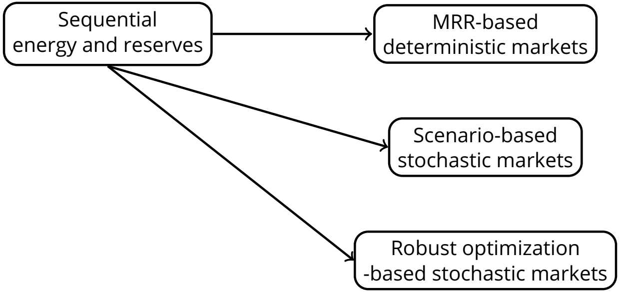

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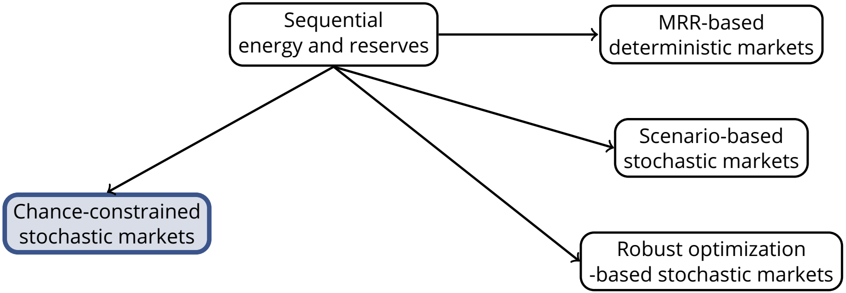

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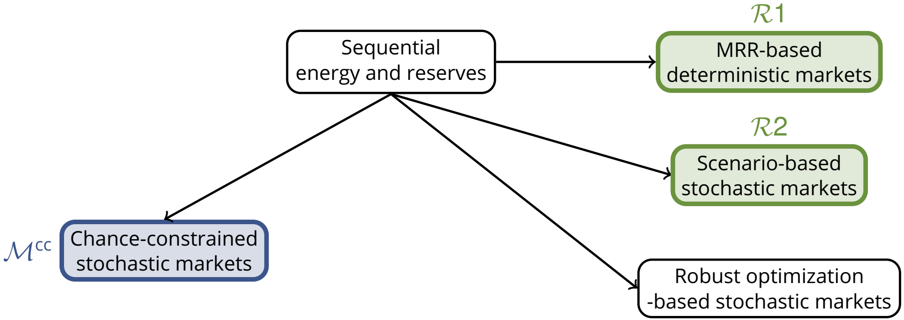
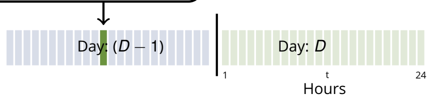
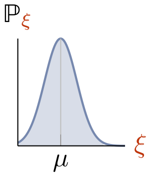


A two-commodity chance-constrained electricity market

Dayahead market-clearing
Commodity 1: **energy**
Commodity 2: **adjustment policy** (α)





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$\sum_i \alpha_i = 1$

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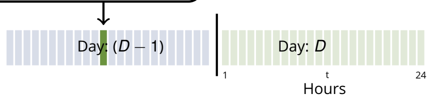

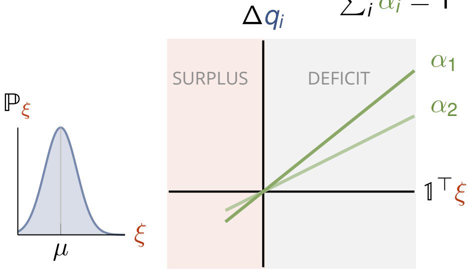

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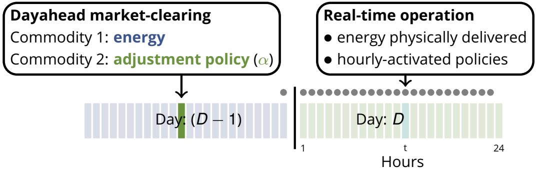

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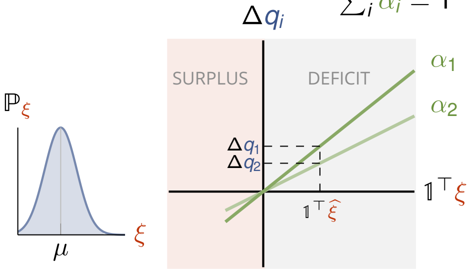

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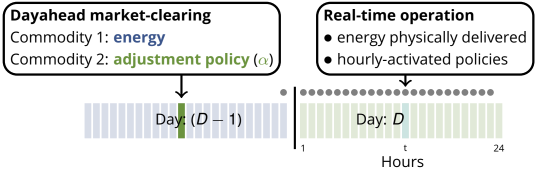

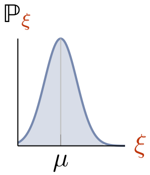

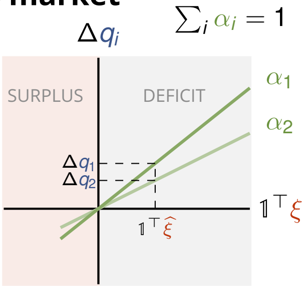

- **SOC reformulation of chance constraints:**

Chance constraint:

$$\mathbb{P}_\xi(\underbrace{q_i + \alpha_i(\mathbb{1}^\top \xi)}_{\Delta q_i} \leqslant \overline{Q}_i) \geqslant (1-\varepsilon)$$

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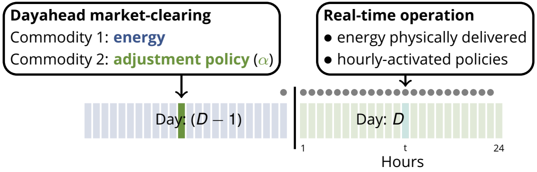

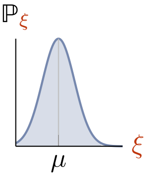

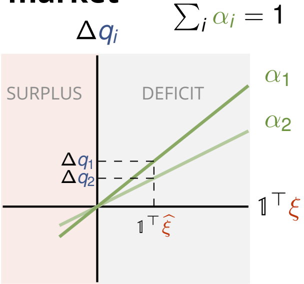

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→

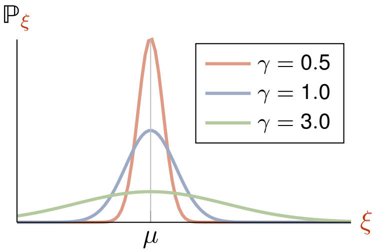
Deterministic SOC constraint:

$$r_\varepsilon \|\mathbf{X}\mathbb{1}\ \alpha_i\| \leqslant \overline{Q}_i - q_i - \mathbb{1}^\top \boldsymbol{\mu}\ \alpha_i$$

where $\boldsymbol{\mu}, \mathbf{X}$ are mean & covariance and r_ε is a safety parameter.

Adjustment policies → Endogenous pricing of flexibility

- Forecast errors: **Gaussian** distribution
- Parameter γ: **variance** of test vs. model distribution



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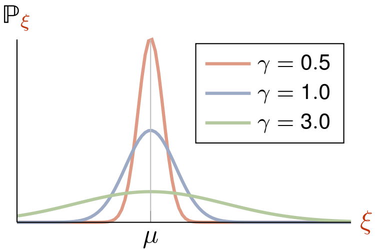

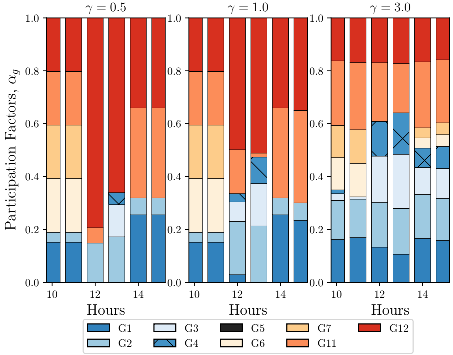

Numerical experiments: 24-node electricity system

- $\mathcal{M}^{cc}$: conic chance-constrained market

- Linear benchmarks
  - $\mathcal{R}1$: deterministic MRR-based market
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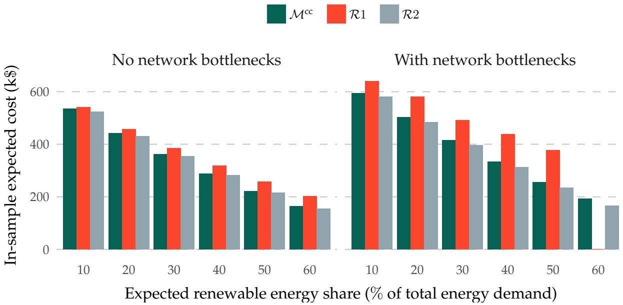

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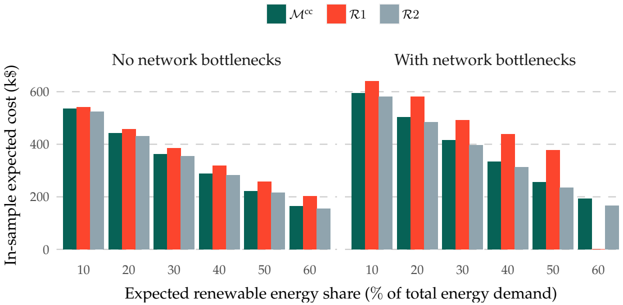
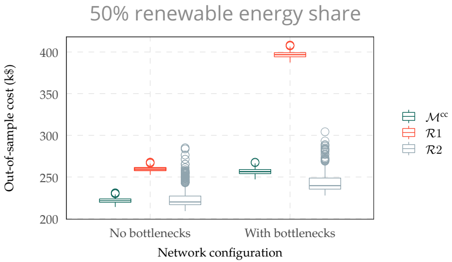
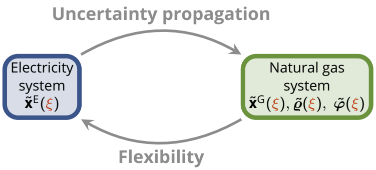




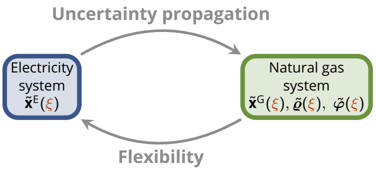
Outline

Stochastic electricity and gas dispatch



Stochastic electricity and gas dispatch

$$\min_{\substack{\tilde{\mathbf{x}}^{\mathrm{E}},\, \tilde{\mathbf{x}}^{\mathrm{G}} \\ \tilde{\varrho},\, \tilde{\varphi}}} \quad \max_{\mathbb{P}_\xi \in \mathcal{P}} \quad \mathbb{E}^{\mathbb{P}_\xi}\left[\sum_{t\in\mathcal{T}} \left(\sum_{i\in\mathcal{I}} c_i^{\mathrm{E}}(\tilde{x}_{it}^{\mathrm{E}}) + \sum_{k\in\mathcal{K}} c_k^{\mathrm{G}}(\tilde{x}_{kt}^{\mathrm{G}}) \right) \right]$$



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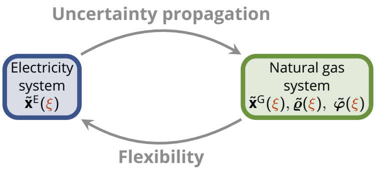

- **Distributionally-robust** chance constraints
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  - robust against **worst-case probability distribution**

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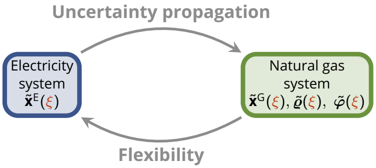

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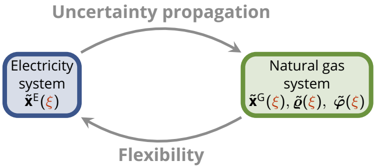

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- **Computationally-intractable**, semi-infinite program:
  1. analytical network response to uncertainty unavailable
  2. non-convex robust joint chance constraints
  3. nonlinear and non-convex state dynamics

Towards computational tractability

Moment-based ambiguity set:

$$\mathcal{P} = \{\mathbb{P}_{\xi} \in \mathcal{P}^0(\mathbb{R}^{WT}) \ : \ \mathbb{E}^{\mathbb{P}_{\xi}}[\xi] = \mu, \ \mathbb{E}^{\mathbb{P}_{\xi}}[\xi\xi^\top] = \Sigma\}$$

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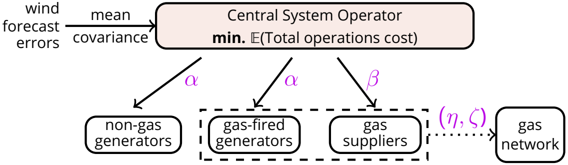

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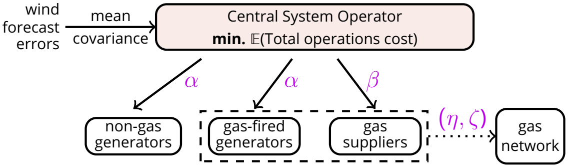

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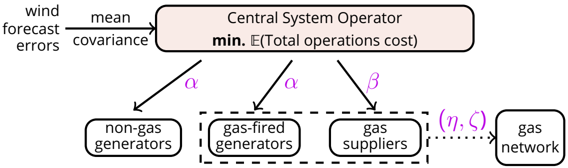
Towards computational tractability



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3. Convexification of **non-convex quadratic** gas flow equations $\mathcal{W}(\tilde{\varrho}_t, \tilde{\varphi}_t) = \mathbb{0}$ by:
   - A. Convex relaxations
   - B. Linearization

Approach A: Using convex relaxations

- Assume **fixed directions** for gas flows & **lossless pressure regulation**

Stochastic quadratic equality:

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Zero-order uncertainty coefficients:
$\mathcal{W}_0(\varrho_t, \varphi_t) = \mathbb{0}$

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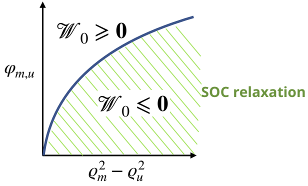

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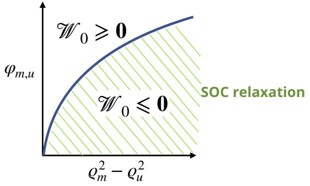

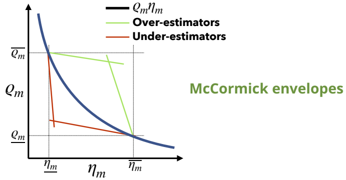

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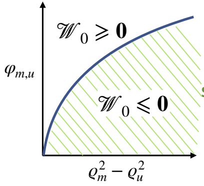
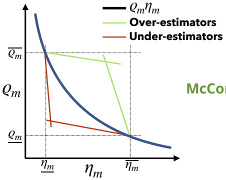


SOC relaxation



McCormick envelopes

- Relaxation tightness impacts **real-time feasibility**

Numerical results: 24-node electricity + 12-node gas system

- Constraints with identical **violation probabilities** $\hat{\varepsilon}$
- 1000 wind forecast scenarios in DK $\rightarrow$ $\boldsymbol{\mu}$ & $\boldsymbol{\Sigma}$ forming **ambiguity set**

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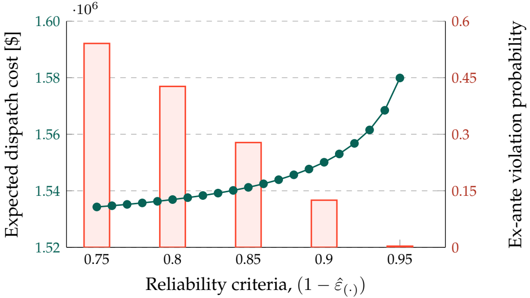

- Trade-off: **expected operations cost** vs. robustness to **uncertainty propagation**

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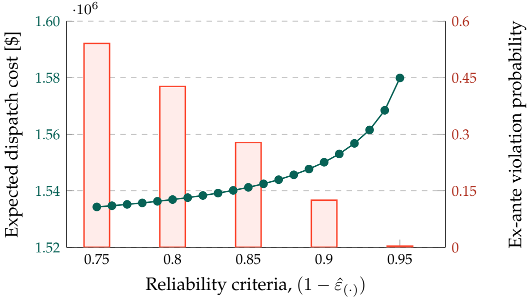
- Constraints with identical **violation probabilities** $\hat{\varepsilon}$
- 1000 wind forecast scenarios in DK $\rightarrow \boldsymbol{\mu}$ & $\boldsymbol{\Sigma}$ forming **ambiguity set**



- Trade-off: **expected operations cost** vs. robustness to **uncertainty propagation**
- Inexact convex relaxations $\rightarrow$ **real-time flow reversals & constraint violations**

Approach B: Using linearization

- **First-order Taylor series expansion** of non-convex gas flow equation:
  1. gas flow directions not fixed
  2. lossy, controllable **pressure regulation** $\tilde{\kappa}$ by compressors & valves
  3. squared pressures, $\tilde{\pi} = \tilde{\varrho}^2$
  4. gas injection & pressure regulation **control policies**: $\tilde{\mathbf{x}}^{G} = \mathbf{x}^{G} + (\mathbb{1}^{\top}\xi)\beta$, $\tilde{\kappa} = \kappa + (\mathbb{1}^{\top}\xi)\gamma$

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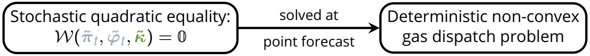

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State variables uncertainty response model

Uncertainty response of state variables is **implicitly** affine in control inputs, i.e.,

$$\tilde{\pi}(\boldsymbol{\xi}) = \pi + \underbrace{\breve{\mathbf{G}}_2(\boldsymbol{\beta} - \hat{\mathbf{G}}_3\boldsymbol{\gamma} - \mathrm{diag}[\mathbb{1}])}_{\eta}\boldsymbol{\xi}\ , \quad \tilde{\varphi}(\boldsymbol{\xi}) = \varphi + \underbrace{(\grave{\mathbf{G}}_2(\boldsymbol{\beta} - \mathrm{diag}[\mathbb{1}]) - \grave{\mathbf{G}}_3\boldsymbol{\gamma})}_{\zeta}\boldsymbol{\xi}$$

Analytical uncertainty response → stochastic gas market design

- Enforce **operational limits** on state variables, e.g., nodal pressure upper bound

$$\mathbb{P}_{\xi}\left[\tilde{\pi}_n(\xi) \leqslant \overline{\pi}_n\right] \geqslant 1-\varepsilon \;\Rightarrow\; r_{\varepsilon}\underbrace{\left\|\mathbf{X}[\breve{\mathbf{G}}_2(\beta-\hat{\mathbf{G}}_3\gamma-\mathrm{diag}[\mathbb{1}])]_n^{\top}\right\|}_{\text{pressure standard deviation}} \leqslant \overline{\pi}_n-\pi_n$$

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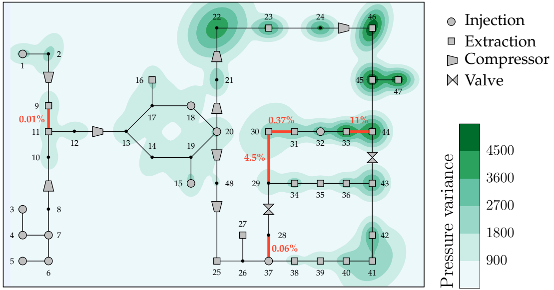
- **Uncertainty-** & **variance-aware** control policies $\rightarrow$ **stochastic SOCP** gas market
- Pricing based on **conic duality** $\rightarrow$ agents have **multiple revenue streams**:
  1. nominal balance
  2. network congestion
  3. recourse balance
  4. variance regulation

Mitigating impacts of uncertainty propagation

Variance-agnostic control policies



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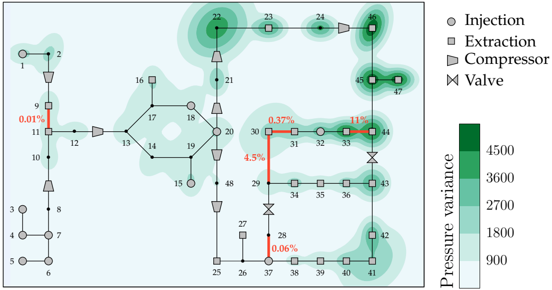
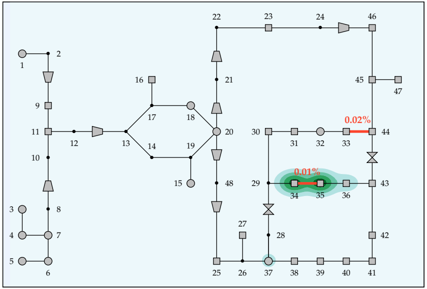


Variance-aware control policies



Outline

Conclusions

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  - nonlinearities in assets, network, and uncertainty models
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  - conic pricing scheme incentivizes uncertainty & variance mitigation services

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   - financial contracts for network flexibility
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3. From centralized coordination to **decentralized or local coordination**
   - local data sharing to improve payoffs and harness cross-carrier flexibility

Thank you for listening.