On Wind Farm Operation with Third-Party Storage

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Abstract—A major challenge in power system operation is the integration of renewable energy in-feed in large scale. Currently, the responsibility to cope with uncertainty in power injection is transferred to a central authority, i.e. the system operator, while renewable energy in-feed is supported via a tariff system. In this paper we propose market participation of wind farms in combination with a third-party energy storage. A novel concept of storage capacity reservation is presented, where the wind power producer hedges unfavorable wind power realizations with a third-party storage. In a day-ahead scheduling stage, profit maximizing bids for the day-ahead market are stated incorporating costs of storage reservation. During an intra-day stage, the storage device backs up the wind power producer by tracking its day-ahead market bids. In a simulation study we show that after the consideration of the costs of storage reservations and storage operation, the proposed model can lead to profitable operation of wind power plants while minimizing the profit variability.

Index Terms—Electricity Markets, Renewable Energy Infeed, Storage

I. INTRODUCTION

The integration of fluctuating renewable energy sources (RES), which are characterised by their high variability in power injection, is a major challenge in power system operation. In electric power systems a balance between generation and consumption has to be remained. Therefore, on the one hand, fluctuations of subsidized RES have been compensated primarily by ancillary services provided by flexible generation sources. Thus, the integration of high shares of RES into the electricity grid is expected to result in higher costs of power system operation [1]. On the other hand, current electricity market frameworks mandate the submission of generation schedules in the form of price/quantity bids prior to actual power generation. Differences between the contracted and actual production lead to imbalance payments, where generators are penalized for deviations from their scheduled production plan. Therefore, the rising costs of system operation can be mitigated if wind power plants have to participate in the energy markets and are obliged to remain their announced schedule. However, RES face difficulties in matching their schedules due to imperfect wind forecasts, which makes this option very costly for the wind farm operators.

In this paper we propose a framework in which wind farms bid into an electricity market and contract a storage provider to compensate for deviations between the forecast and the actual wind power realization. The model comprises a day-ahead (DA) scheduling stage, where storage capacity can be reserved, and an intra-day (ID) stage where deviations by the wind producer are compensated through the storage device. By avoiding costly imbalance payments the framework may enable profitable market participation of a wind farm. In the DA stage, optimal hourly bids are determined. A novel concept for storage capacity reservation, which acts as a hedging mechanism against unfavorable wind power realizations, is introduced. Storage devices need not to be owned by the wind power plant. The ID stage of the model involves the operation of the storage device. The increase in accuracy of wind power forecasts with shrinking forecast horizon is utilized by Model-Predictive-Control (MPC). The control strategy also decides on adjustment bids for the ID market.

The contributions of this paper are threefold: First, we propose a model for strategic bidding of wind power plants to minimize the risk of imbalance penalties. Second, we quantify the benefits of storage as compared with the situation when a wind power plant participates in electricity markets on its own. Third, we estimate the incremental cost of using storage resources for mitigation of power imbalances in the electricity grid caused by wind power plants.

There exists a rich literature on dealing with wind uncertainty in power systems. Stochastic programming, incorporating uncertain wind power forecasts and market prices as stochastic variables, may be used to develop models which result in optimal bidding strategies for a wind power producer participating in electricity spot markets on its own e.g. [2], [3], [4], [5]. The design of financial products for hedging is proposed in e.g. [6]. Various other approaches for the operation of a storage device in combination with a wind power plant are presented in e.g. [7], [8], [9], [10]. These works either do not consider a DA/ID market framework or do a joint optimization of a wind farm with storage which assumes that the storage be operated from the same stakeholder.

The remainder of this paper is organized as follows. Section II describes the proposed DA scheduling methodology. Section III describes the ID market operation. Section IV explains the simulation setup and in section V results are presented. Section VI concludes the paper and outlines further work.

II. DAY-AHEAD MARKET OPERATION

A. Uncertainty in Wind Power Injection

The hourly output of a wind farm is considered to be a random variable X with a continuous probability distribution function, P(X), such that:

$$P(X) = f(\mu, \sigma, \rho, \cdots), \tag{1}$$

where μ , σ , ρ , \cdots represent the parameters of the distribution function, f, namely, expected value, standard deviation, skewness, etc.. This model allows the use of any continuous probability distribution function (pdf) for characterizing the uncertainty in wind power forecasts, provided that (a) its pdf and cumulative distribution function (cdf) can be expressed analytically, and (b) the inverse cumulative distribution function, or quantile function exists. In our model wind power injection is characterized by a normal distribution function [11], where the point forecast is the expected value. The standard deviation is expressed as a function of the ratio between the point forecast value and the rated capacity of the wind power plant. The function can be determined via fitting methods and the analysis of historical data.

B. Problem Formulation

While deciding on bids for the DA market, the wind power producer must consider an uncertainty in wind power production. The assumptions for the bidding in the DA market are that there are capacity limits of the storage device, and the prices for the DA market are assumed to be deterministic and known. The optimization problem is defined as:

$$\max_{(z,\alpha,S)\forall h} \qquad \rho \sum_{h=1}^{24} (\Lambda_h^\mathsf{T} z_h) - (1-\rho) |\sum_{h=1}^{24} (\lambda_h^\mathsf{S} S_h)|$$

$$\text{subject to}, \forall h, \quad z_h = \begin{bmatrix} B_h & C_h^u & C_h^d & R_h & (I_h^u + I_h^d) \end{bmatrix} \tag{2}$$

$$0 < \alpha_h < 1 \tag{3}$$

$$b_h^u = \Phi^{-1} \left(\frac{1 + \alpha_h}{2} \right) \tag{4}$$

$$b_h^d = \Phi^{-1} \left(\frac{1 - \alpha_h}{2} \right) \tag{5}$$

$$b_h^d \le B_h \le b_h^u \tag{6}$$

$$C_h^u = b_h^u - B_h \tag{7}$$

$$C_h^d = B_h - b_h^d \tag{8}$$

$$R_h = \begin{cases} \frac{|-(B_1)^2|}{2\delta}, \ \forall h = 1\\ \frac{|(B_{h-1})^2 - (B_h)^2|}{2\delta}, \ \forall h \neq 1 \end{cases}$$
(9)

$$I_h^u = \mathbb{E}(X \mid X_h > b_h^u) - b_h^u$$
 (10)

$$I_h^d = b_h^d - \mathbb{E}(X \mid X_h < b_h^d) \tag{11}$$

$$S_h = B_h - W_h \tag{12}$$

where

 I_h^u, I_h^d : Expected imbalances (MW),

 b_h^u, b_h^d : Upper and lower bounds (MW) for DA mar-

ket bid B_h ,

 C_h^u, C_h^d : Up (charge) and down (discharge) storage

capacity reservation (MW),

 $\lambda_h^{\mathrm{C}_u}, \lambda_h^{\mathrm{C}_d}$: Storage reservation costs (\in /MW) in the up

(charging) and down (discharging) directions

for the hour h.

h : Hours of the day $(1,2,\dots,24)$,

 X_h : Random variable for wind power (MW) outcome, W_h : Available point forecast for wind power (MW),

 B_h : Bid (MW) in DA market,

 α_h : Risk tolerance factor which decides the position

of bounds, such that $\alpha_h \in [0, 1)$,

 R_h : Ramping energy (MWh) between hours,

 S_h : Expected use of storage (MW), Φ^{-1} : Quantile function or inverse cdf,

o : Fixed parameter to control the strictness of

energy-neutrality requirement in DA bids,

 δ $\,\,$: Maximum change in bids allowed in bids be-

tween subsequent hours (MW/hour),

 λ_h^{DA} : DA market price (\in /MW) forecast,

 λ_h^R : Penalty for ramping energy (\in /MWh),

 λ_h^{I} : Penalty for imbalances (\in /MW),

 λ_h^{S} : Cost for storage operation (\in /MW), Λ_h : $-\left[-\lambda_h^{DA} \ \lambda_h^{C_u} \ \lambda_h^{C_d} \ \lambda_h^{R} \ \lambda_h^{I}\right]$.

The objective function in the optimization problem comprises two terms which are related to each other via $\rho \in [0,1]$. In the first term, the bids for the DA market are decided based on a trade-off between the expected revenues from the DA market and the costs associated with it. These costs include the reservation costs for storage capacity, $\lambda_h^{C_h^u}$ and $\lambda_h^{C_h^u}$, and the costs of imbalances penalties, λ_h^I . The second term in the objective function incentivizes energy-neutrality of the bids during the day. It relates bids in the DA market to the realistic operation of storage device during the next day. It implies that the expected energy for charging the storage device should be as close as possible to the expected energy of discharging. This is a crucial consideration for achieving the decoupling of the storage device from the wind power plant. Thus, if ρ is chosen equal to 1, the energy neutrality term is eliminated from the optimization problem.

Constraint (3) provides bounds for the choice of α_h . The decision variable α_h indicates the preference to place bids, B_h , different from the point forecast, W_h . Variable α_h defines an interval for the bounds $[b_h^d, b_h^u]$, within which the bid for the hour is chosen. In the absence of storage, or if the cost of storage is high, α_h is close to zero and the bounds are close to the point forecast, W_h . Since W_h represents the expected value of a random wind power realization, X_h , there is a risk of paying for imbalances involved with bids different from W_h . The bounds b_h^d and b_h^u , defined in equality constraints in (4) and (5), are calculated from the inverse cdf, or quantile function of the probability distribution of X_h for a chosen value of α_h . The bounds may be symmetrical around W_h , depending on whether the probability distribution $P(X_h)$ is

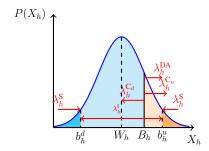


Fig. 1: Trade-offs in the DA optimization problem which influence the choice of the optimal bid B_h .

symmetrical around W_h . Constraints (7) and (8) determine the storage capacity, C_h^u and C_h^d , to be reserved. The storage capacity reservations are determined by the distance of the bid B_h from the corresponding upper bound b_h^u and lower bound b_h^d respectively.

In case of high shares of wind energy, flexible thermal units in the power systems may need to accommodate frequent changes to their scheduled production. This leads to higher costs involved with frequent ramping of thermal units. Therefore, the system operator may impose restrictions on wind power producers with regards to changes between subsequent hourly bids. Constraint (9) penalizes extreme changes in the bids between subsequent hours. Constraints (10) and (11) penalize expected imbalances. Imbalances are expected for wind power realizations which lie outside the interval $[b_h^d, b_h^u]$, where the reserved storage capacities are not enough to accommodate the difference between bid B_h and the wind outcome X_h . Constraint (12) defines the expected storage use for each hour during the actual production in the next day. Fig. 1 demonstrates the trade-offs in the optimization problem and the choice of optimal bid B_h within the bounds defined by b_h^d and b_h^u .

III. INTRADAY MARKET OPERATION

An ID market with continuous trading is considered. Therefore, the risk that bids in a hourly ID market may not be cleared because of a lack of a suitable counter-party are reflected in terms of higher costs associated with the bids in this market. This provides a preference for operating the storage, whenever feasible. The objectives of the ID market operation stage are first that the contracted production schedule received from the DA market clearing should be tracked, such that imbalance penalties are minimized. Second, the storage device should be operated within the limits of reserved capacity obtained from the results of the DA scheduling stage. Third, suitable adjustment bids for the ID markets should be prepared. These adjustment bids have to mitigate the imbalances occurring when the power reservations for storage or the energy content of the storage device are not sufficient. Further, the adjustment bids maintain the daily energy neutrality of the storage device.

A MPC based operation framework is used. MPC is a broad term for any control formulation where an optimal control trajectory for a given system is obtained through solving a cost minimization problem. The first step involved in MPC is the mathematical modeling of the underlying system. State space models are most commonly used for this purpose. Ref. [12] provides a detailed discussion on designing aspects of MPC systems and their implementation. In each hour, the optimization problem is solved to obtain the charging/discharging schedule for the storage device for the next hour along with the ID adjustment bids. The scheduled operation is undertaken irrespective of the wind realization in the next hour. This is because the storage device is decoupled from the wind power producer and hence it doesn't provide real-time imbalance mitigation support. The ID market optimization problem is defined as follows:

$$\min_{u} \sum_{N=24}^{1} \left(\sum_{h=(25-N)}^{24} \right) \\
\left[e(h)^{T} \Omega_{\text{imb}} e(h) + u(h)^{T} \Omega_{\text{con}} u(h) + w(h)^{T} \Omega_{\text{ex}} w(h) \right])$$
(13)

s.t.,
$$\forall h, \quad e(h) = (y(h) - P_{\text{sched}}(h))$$
 (14)

$$u_{1,h} \cdot u_{2,h} = 0 \tag{15}$$

$$u_{3,h} \cdot u_{4,h} = 0 \tag{16}$$

$$0 \le u_{1,h} \le C_h^{\mathsf{d}} \tag{17}$$

$$0 \le u_{2,h} \le C_h^{\mathsf{u}} \tag{18}$$

$$0 \le u_{3,h} \le P_{\mathsf{R}}^{\mathsf{max}} \tag{19}$$

$$0 \le u_{4,h} \le P_{\mathcal{S}}^{\text{max}} \tag{20}$$

$$(E_{min} - \omega(h)) < x(h) < (E_{max} + \omega(h)) \quad (21)$$

$$w(h) > 0 \tag{22}$$

$$\begin{bmatrix} E_{S}(h+1) \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix} \begin{bmatrix} E_{S}(h) \end{bmatrix} + \begin{bmatrix} \frac{-\tau}{\eta_{D}} & \tau \cdot \eta_{C} & 0 & 0 \end{bmatrix} \begin{bmatrix} P_{D}(h) \\ P_{C}(h) \\ P_{B}(h) \\ P_{S}(h) \end{bmatrix}$$
(23)

$$P_{G}(h) = \begin{bmatrix} 1 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} P_{D}(h) \\ P_{C}(h) \\ P_{B}(h) \\ P_{S}(h) \end{bmatrix} + \begin{bmatrix} 1 \end{bmatrix} P_{W}(h)$$
 (24)

and

 $\begin{array}{l} P_{\rm sched}(h) \\ C_h^{\rm d}, C_h^{\rm u} \end{array}$ DA scheduled power (MW) in hour h,

Discharging/Charging capacity reserva-

tions (MW),

 $P_{\mathrm{B}}^{\mathrm{max}}, P_{\mathrm{S}}^{\mathrm{max}}$ Maximum adjustment bids (MW) allowed

in the ID market,

 E_{min}, E_{max} : Minimum and maximum operating limits

allowed for energy content (MWh) in the

storage device,

 $P_{\mathbf{D}}(h), P_{\mathbf{C}}(h)$: Discharging/Charging power from/into storage device,

 $P_{\mathbf{B}}(h), P_{\mathbf{S}(h)}$: Adjustment bid made in the ID market,

y(h): Controller output, in-feed to the grid (MW),

Error (MW) in tracking the DA scheduled power,

x(h): Energy Content (MWh) of storage device at the

start.

u(h): Control vector ($[P_D \quad P_C \quad P_B \quad P_S]$),

v(h): Best available wind forecast, w(h): Auxiliary variable (MWh),

 Ω_{imb} : Cost for errors in meeting schedule,

 $\Omega_{\rm con}$: Costs for control variables,

 Ω_{ex} : Cost for exceeding storage operating limits,

 τ : Time of operation (1 hour),

 $E_{\rm S}(h)$: Energy stored in storage device at the start,

 $P_{G}(h)$: Power in-feed into the grid from wind and storage

ensemble,

 $P_{W}(h)$: Best available forecast for wind power,

 $\eta_{\rm D}, \eta_{\rm C}$: Discharging/Charging efficiency of the storage

device.

In addition to the penalty for errors in tracking the DA bids, Ω_{imb} , the objective function includes the cost for the control vector, Ω_{con} , and the cost of exceeding the storage operating limits, $\Omega_{\rm ex}$. The risk associated with a continuous trading mechanism in an ID market is accounted for in the cost vector via a higher preference for storage operation over the ID market bidding. In order to maintain the energy-neutrality of the storage device, the adjustment bids are designed in the ID market such that the net earnings from participation in hourly ID markets over the optimization horizon are maximized. This is accomplished through a higher preference for buying bids in the ID market in hours when market prices are expected to be low and vice-versa. Constraint (21) allows the violation of the operating limits of the energy content of storage at high costs. Constraint (15) ensures that the storage device is not charged and discharged at the same hour. Constraint (16) allows only buying or selling bids in every hour. Constraints (17)-(20) provide bounds on the control variables. The hourly storage capacity reservations received from the DA scheduling stage are enforced as upper bounds for storage operation. The bounds on maximum ID adjustment bids are pre-defined. The change in energy content of the storage device with time is quantified in (23), while the output from the system is defined by (24).

IV. SIMULATION SETUP

To validate the performance of the model, four cases are defined with regards to dealing with uncertainty in wind power availability. Table I summarizes the characteristics of these cases. Simulations for a time period of 10 days are

Case	Description
Perfect Forecast (PF)	Perfect Wind Forecasts, No storage requirement,
	No ID market participation
Only ID (OID)	Uncertain wind, No storage access, With ID
	market participation
Only Storage (OS)	Uncertain wind, With storage, No ID market
	participation
Storage+ID (SID)	Uncertain wind, With storage access, With ID
	market participation

TABLE I: Summary of the various cases.

performed for each of the cases. In all cases, a wind power farm is considered with an assumed installed capacity of 50 MW. The size of storage device of 20 MWh. Normalized hourly values of wind power forecasts and actual injection

for Germany over the years 2007-2011 are used. The model parameters used in the simulations are described [13]. The problem in the DA scheduling stage is a constrained non-linear optimization problem. The optimization problem for the ID market is a quadratic program with mixed integer constraints. Both problems are formulated in MATLAB using the YALMIP toolbox and the latter is solved using IBM's ILOG CPLEX solver [14] [15].

V. RESULTS

A. End-of-day Profits

We evaluate the net profits earned at the end of each day for the various cases discussed before. Profits at the end of each day d are defined as:

$$Profits^{d} = Rev_{DA}^{d} + Rev_{ID}^{d} - Imb^{d} - Stor^{d},$$
 (25)

where $\operatorname{Rev}_{\mathrm{DA}}^d$ and $\operatorname{Rev}_{\mathrm{ID}}^d$ are the revenues of DA and ID market operation respectively. Imb^d and Stor^d are the costs of imbalances and storage. The end-of-day revenues from the ID market settlements could be positive or negative depending upon the sum of all buying and selling cash flows during the day. The case PF is not associated with either of the

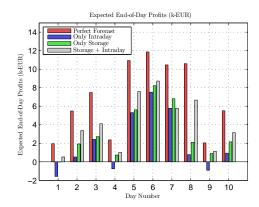


Fig. 2: Expected profits earned at the end of the day for different cases.

two costs stated in (25). The profits in this case act as an upper bound for the possible profits which can be made. Fig. 2 shows that in the OID case, there are days when the expected profits are negative. This occurs for days with low profits even for the PF case, which indicates that a relatively lower total energy volume is bid into the DA market on those days and that attempts to manage imbalances only using ID bids can be risky for such days. On the other hand, using a storage device as in the other two cases, OS and SID, leads to higher profits even after the costs of operating storage have been taken into consideration. The benefit of using a storage device becomes obvious if the variability in the end-of-day profits is evaluated, as shown in Fig. 3. The box-plot shows the descriptive statistics for the end-of-day profits in the OID case, such that edges of the box represent the 25th and 75th percentiles and red-colored ticks inside the boxes represent the median value. The variability of profits indicated by whiskers around the box represents 99% of occurrences. It can be observed that in the OID case, the chances of making less

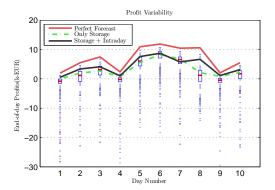


Fig. 3: Variability of the profits earned at the end of the day for various cases. The boxplot shows the statistical distribution of profits for the OID case.

profits and even negative profits is quite high. Thus, it can be concluded that even after including the costs of storage operation, OS and SID, profits in each day can be achieved, without a downside risk in profit variability.

B. Cost of Storage for Imbalance Mitigation

For the OID and SID cases, Fig. 4 shows the average cost of storage operation for incremental values of % imbalances mitigated for the 10 days of simulation. Imbalances mitigated in percent is defined as:

$$Imb = 100 \times \left(\frac{\sum_{h=1}^{24} \text{Deviations from schedule [MW]}}{\sum_{h=1}^{24} \text{ Scheduled power [MW]}}\right)$$
 (26)

These curves are obtained by changing the energy rating of storage device (MWh) such that a saturation in % imbalances mitigated could be reached. Fig. 4 shows that the Storage+ID case is preferable over the OS because for the same cost of storage, the percentage of imbalances mitigated in the former is higher. As reasoned before, the absence of the option to participate in ID markets leads to significantly higher imbalances in the OS case.

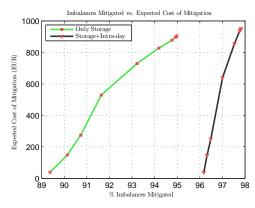


Fig. 4: Expected costs of using storage for imbalance mitigation for Only Storage and Storage+Intra-day cases.

VI. CONCLUSIONS

This paper proposed a new method to assess the combination of wind farms with storage. The analytical properties of probability functions have been used to determine the optimal size of storage capacity reservation. Subsequently, Model Predictive Control has been used to even out deviations from the wind farm in the intraday market. It has been shown that the combination of intraday market operation and contracting third party storage is a profitable options for wind sites. In case of intraday only operation wind farms can make losses through imbalance penalties. Future research includes extension of the model where the wind farm is not only a price taker but a price maker, and the inclusion of a grid.

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